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General Lower Bounds for $b \to d$ Penguin Processes

Robert Fleischer[a] **and Stefan Recksiegel**[b]

[a] *Theory Division, Department of Physics, CERN, CH-1211 Geneva 23, Switzerland*

[b] *Physik Department, Technische Universität München, D-85748 Garching, Germany*

Abstract

For the exploration of flavour physics, $b \to d$ penguin processes are an important aspect, with the prominent example of $\bar{B}^0_d \to K^0 \bar{K}^0$. We recently derived lower bounds for the CP-averaged branching ratio of this channel in the Standard Model; they were found to be very close to the corresponding experimental upper limits, thereby suggesting that $\bar{B}^0_d \to K^0 \bar{K}^0$ should soon be observed. In fact, the BaBar collaboration subsequently announced the first signals of this transition. Here we point out that it is also possible to derive lower bounds for $\bar{B} \to \rho\gamma$ decays, which are again surprisingly close to the current experimental upper limits. We show that these bounds are realizations of a general bound that holds within the Standard Model for $b \to d$ penguin processes, allowing further applications to decays of the kind $B^\pm \to K^{(*)\pm} K^{(*)}$ and $B^\pm \to \pi^\pm \ell^+ \ell^-$, $\rho^\pm \ell^+ \ell^-$.

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Robert Fleischer[1] and Stefan Recksiegel[2]

[1] *Theory Division, Department of Physics, CERN, CH-1211 Geneva 23, Switzerland*
[2] *Physik Department, Technische Universität München, D-85748 Garching, Germany*

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For the exploration of flavour physics, $b \to d$ penguin processes are an important aspect, with the prominent example of $\bar{B}^0_d \to K^0\bar{K}^0$. We recently derived lower bounds for the CP-averaged branching ratio of this channel in the Standard Model; they were found to be very close to the corresponding experimental upper limits, thereby suggesting that $\bar{B}^0_d \to K^0\bar{K}^0$ should soon be observed. In fact, the BaBar collaboration subsequently announced the first signals of this transition. Here we point out that it is also possible to derive lower bounds for $\bar{B} \to \rho\gamma$ decays, which are again surprisingly close to the current experimental upper limits. We show that these bounds are realizations of a general bound that holds within the Standard Model for $b \to d$ penguin processes, allowing further applications to decays of the kind $B^\pm \to K^{(*)\pm}K^{(*)}$ and $B^\pm \to \pi^\pm\ell^+\ell^-$, $\rho^\pm\ell^+\ell^-$.

Keywords: rare B decays, $b \to d$ penguin processes

Despite the tremendous progress at the B factories, we still have few insights into the rare decays that are mediated by $b \to d$ penguin topologies, which represent a key element in the testing of the quark-flavour sector of the Standard Model (SM) that is described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1]. An important example is the decay $\bar{B}^0_d \to K^0\bar{K}^0$, which originates from $b \to d\bar{s}s$ flavour-changing neutral-current (FCNC) processes. Within the SM, these are governed by QCD penguin-like topologies, so that we may write

$$A(\bar{B}^0_d \to K^0\bar{K}^0) \propto \mathcal{P}_{tc}^{KK}\left[1-\rho_{KK}e^{i\theta_{KK}}e^{-i\gamma}\right], \quad (1)$$

where γ is the usual angle of the unitarity triangle of the CKM matrix, whereas $\mathcal{P}_{tc}^{KK}$ and $\rho_{KK}e^{i\theta_{KK}}$ are CP-conserving hadronic parameters. The latter quantities not only depend on the penguin topologies with internal top-quark exchanges, but are also expected to be affected significantly by those with internal up- and charm-quark exchanges [2], containing also long-distance rescattering effects [3]. Consequently, it seems essentially impossible to calculate $\mathcal{P}_{tc}^{KK}$ and $\rho_{KK}e^{i\theta_{KK}}$ in a reliable manner from first principles, despite theoretical progress [4].

In a recent paper [5], we addressed this problem, and pointed out that *lower* bounds for the CP-averaged branching ratio $\mathrm{BR}(B_d \to K^0\bar{K}^0)$ can be derived in the SM. To this end, we assume that $\gamma = (65 \pm 7)^\circ$, as in the SM [6], consider ρ_{KK} and θ_{KK} as "unknown" parameters, i.e. vary them within their whole physical range, and fix $|\mathcal{P}_{tc}^{KK}|$ with the help of the $SU(3)$ flavour symmetry of strong interactions. The strategy developed in [7] offers the following two avenues, using data for

i) $B \to \pi\pi$ ($b \to d$) decays:

$$\mathrm{BR}(B_d \to K^0\bar{K}^0)_{\min} = \Xi^K_\pi \times \left(1.39\,^{+1.54}_{-0.95}\right) \times 10^{-6}, \quad (2)$$

ii) $B \to \pi K$ ($b \to s$) decays, complemented by the $B \to \pi\pi$ system to determine a small correction:

$$\mathrm{BR}(B_d \to K^0\bar{K}^0)_{\min} = \Xi^K_\pi \times \left(1.36\,^{+0.18}_{-0.21}\right) \times 10^{-6}. \quad (3)$$

Here we have included factorizable $SU(3)$-breaking corrections, making their impact explicit through

$$\Xi^K_\pi = \left[\frac{f_0^K}{0.331}\frac{0.258}{f_0^\pi}\right]^2; \quad (4)$$

the numerical values for the $B \to K, \pi$ form factors $f_0^{K,\pi}$ refer to a recent light-cone sum-rule analysis [8]. Comparing (2) and (3) with the experimental *upper* bound of $\mathrm{BR}(B_d \to K^0\bar{K}^0) < 1.5\times10^{-6}$ (90% C.L.), we concluded that $\bar{B}^0_d \to K^0\bar{K}^0$ should soon be observed.

In fact, the BaBar collaboration subsequently reported the first signals, with the CP-averaged branching ratio $\mathrm{BR}(B_d \to K^0\bar{K}^0) = (1.19\,^{+0.40}_{-0.35} \pm 0.13) \times 10^{-6}$ [9]. This is a very exciting measurement, as it establishes – for the first time – a $b \to d$ penguin process. The consistency of the BaBar result [9] with (2) and (3) is a first successful test of the SM description of the $b \to d\bar{s}s$ FCNC processes, although the current uncertainties are still large; using the most recent data [10, 11], the numerical factors in (2) and (3) are modified as $(1.15\,^{+1.13}_{-0.77}) \times 10^{-6}$ and $(1.37\,^{+0.16}_{-0.20})\times10^{-6}$, respectively. More powerful tests will be possible in the future, where also the CP-violating $B_d \to K^0\bar{K}^0$ asymmetries will play a key rôle [5]; a specific new-physics analysis within the framework of supersymmetry was very recently performed in [12].

Another important tool to explore the $b \to d$ penguin sector is provided by $\bar{B} \to \rho\gamma$ modes (for recent analyses, see [13, 14]). The current experimental picture of their CP-averaged branching ratios is given as follows:

$$\mathrm{BR}(B^\pm \to \rho^\pm\gamma) < \begin{cases} 1.8 \times 10^{-6} & \text{(BaBar [15])} \\ 2.2 \times 10^{-6} & \text{(Belle [16])} \end{cases} \quad (5)$$

$$\mathrm{BR}(B_d \to \rho^0\gamma) < \begin{cases} 0.4 \times 10^{-6} & \text{(BaBar [15])} \\ 0.8 \times 10^{-6} & \text{(Belle [16]),} \end{cases} \quad (6)$$

where the upper bounds are at the 90% confidence level. In the SM, these decays are described by a low-energy

effective Hamiltonian of the following structure [17]:

$$\mathcal{H}_{\rm eff}^{b\to d\gamma}=\frac{G_{\rm F}}{\sqrt{2}}\sum_{j=u,c}V_{jd}^{*}V_{jb}\left[\sum_{k=1}^{2}C_kQ_k^{jd}+\sum_{k=3}^{8}C_kQ_k^{d}\right]. \quad (7)$$

Here the $Q_{1,2}^{jd}$ denote the current–current operators, whereas the $Q_{3\ldots 6}^{d}$ are the QCD penguin operators, which govern the decay $\bar{B}_d^0\to K^0\bar{K}^0$ together with the penguin-like contractions of $Q_{1,2}^{cd}$ and $Q_{1,2}^{ud}$. On the other hand,

$$Q_{7,8}^{d}=\frac{1}{8\pi^2}m_b\bar{d}_i\sigma^{\mu\nu}(1+\gamma_5)\left\{eb_iF_{\mu\nu},\, g_{\rm s}T_{ij}^{a}b_jG_{\mu\nu}^{a}\right\} \quad (8)$$

are the electro- and chromomagnetic penguin operators.

Using (7) and the Wolfenstein parametrization of the CKM matrix [18], generalized to the next-to-leading order in $\lambda=0.224$ [19], we may write

$$A(\bar{B}\to\rho\gamma)=c_\rho\lambda^3A\mathcal{P}_{tc}^{\rho\gamma}\left[1-\rho_{\rho\gamma}e^{i\theta_{\rho\gamma}}e^{-i\gamma}\right], \quad (9)$$

where $c_\rho=1/\sqrt{2}$ and 1 for $\rho=\rho^0$ and $\rho^{\pm}$, respectively, $A=|V_{cb}|/\lambda^2$, $\mathcal{P}_{tc}^{\rho\gamma}\equiv\mathcal{P}_{t}^{\rho\gamma}-\mathcal{P}_{c}^{\rho\gamma}$, and

$$\rho_{\rho\gamma}e^{i\theta_{\rho\gamma}}\equiv R_b\left[\frac{\mathcal{P}_{t}^{\rho\gamma}-\mathcal{P}_{u}^{\rho\gamma}}{\mathcal{P}_{t}^{\rho\gamma}-\mathcal{P}_{c}^{\rho\gamma}}\right]. \quad (10)$$

The $\mathcal{P}_{j}^{\rho\gamma}$ are strong amplitudes, which have the following interpretation: $\mathcal{P}_{u}^{\rho\gamma}$ and $\mathcal{P}_{c}^{\rho\gamma}$ refer to the matrix elements of $\sum_{k=1}^{2}C_kQ_k^{ud}$ and $\sum_{k=1}^{2}C_kQ_k^{cd}$, respectively, whereas $\mathcal{P}_{t}^{\rho\gamma}$ corresponds to $-\sum_{k=3}^{8}C_kQ_k^{d}$. Consequently, $\mathcal{P}_{u}^{\rho\gamma}$ and $\mathcal{P}_{c}^{\rho\gamma}$ describe the penguin topologies with internal up- and charm-quark exchanges, respectively, whereas $\mathcal{P}_{t}^{\rho\gamma}$ corresponds to the penguins with the top quark running in the loop. Finally, $R_b\propto|V_{ub}/V_{cb}|$ is one side of the unitarity triangle [19]. Let us note that (9) refers to a given photon helicity. However, the b quarks couple predominantly to left-handed photons in the SM, so that the right-handed amplitude is usually neglected [20]; we shall return to this point below. Comparing (9) with (1), we observe that the structure of both amplitudes is the same. In analogy to $\rho_{KK}e^{i\theta_{KK}}$, $\rho_{\rho\gamma}e^{i\theta_{\rho\gamma}}$ may also be affected by long-distance effects, which represent a key uncertainty of $\bar{B}\to\rho\gamma$ decays [20, 21].

If we replace all down quarks in (7) by strange quarks, we obtain the effective Hamiltonian for $b\to s\gamma$ processes, which are already well established experimentally [11]:

$$\begin{aligned}{\rm BR}(B^{\pm}\to K^{*\pm}\gamma) &= (40.3\pm2.6)\times10^{-6} \quad (11)\\ {\rm BR}(B_d^{0}\to K^{*0}\gamma) &= (40.1\pm2.0)\times10^{-6}. \quad (12)\end{aligned}$$

In analogy to (9), we may write

$$A(\bar{B}\to K^*\gamma)=-\frac{\lambda^3A\mathcal{P}_{tc}^{K^*\gamma}}{\sqrt{\epsilon}}\left[1+\epsilon\rho_{K^*\gamma}e^{i\theta_{K^*\gamma}}e^{-i\gamma}\right], \quad (13)$$

with $\epsilon\equiv\lambda^2/(1-\lambda^2)=0.053$. Thanks to the smallness of ϵ, the parameter $\rho_{K^*\gamma}e^{i\theta_{K^*\gamma}}$ plays an essentially negligible rôle for the $\bar{B}\to K^*\gamma$ transitions.

Let us first focus on the charged decays $B^{\pm}\to\rho^{\pm}\gamma$ and $B^{\pm}\to K^{*\pm}\gamma$. Here we obtain

$$\frac{{\rm BR}(B^{\pm}\to\rho^{\pm}\gamma)}{{\rm BR}(B^{\pm}\to K^{*\pm}\gamma)}=\epsilon\left[\frac{\Phi_{\rho\gamma}}{\Phi_{K^*\gamma}}\right]\left|\frac{\mathcal{P}_{tc}^{\rho\gamma}}{\mathcal{P}_{tc}^{K^*\gamma}}\right|^2H_{K^*\gamma}^{\rho\gamma}, \quad (14)$$

where $\Phi_{\rho\gamma}$ and $\Phi_{K^*\gamma}$ denote phase-space factors, and

$$H_{K^*\gamma}^{\rho\gamma}\equiv\frac{1-2\rho_{\rho\gamma}\cos\theta_{\rho\gamma}\cos\gamma+\rho_{\rho\gamma}^2}{1+2\epsilon\rho_{K^*\gamma}\cos\theta_{K^*\gamma}\cos\gamma+\epsilon^2\rho_{K^*\gamma}^2}. \quad (15)$$

If we apply now the U-spin flavour symmetry of strong interactions to these rare decays [22], which is a subgroup of flavour $SU(3)$ that relates down and strange quarks in the same manner as the conventional isospin symmetry relates down and up quarks, we obtain

$$|\mathcal{P}_{tc}^{\rho\gamma}|=|\mathcal{P}_{tc}^{K^*\gamma}| \quad (16)$$

$$\rho_{\rho\gamma}e^{i\theta_{\rho\gamma}}=\rho_{K^*\gamma}e^{i\theta_{K^*\gamma}}\equiv\rho e^{i\theta}. \quad (17)$$

Although (16) allows us to determine the ratio of the penguin amplitudes $|\mathcal{P}_{tc}|$ in (14) – up to $SU(3)$-breaking effects to be discussed below – we are still left with the dependence on ρ and θ. However, if we keep ρ and θ as free parameters, we may show that $H_{K^*\gamma}^{\rho\gamma}$ satisfies

$$H_{K^*\gamma}^{\rho\gamma}\geq\left[1-2\epsilon\cos^2\gamma+\mathcal{O}(\epsilon^2)\right]\sin^2\gamma, \quad (18)$$

where the term linear in ϵ gives a shift of about 1.9%.

Concerning possible $SU(3)$-breaking effects to (17), they may only enter this tiny correction and are negligible for our analysis. On the other hand, the $SU(3)$-breaking corrections to (16) have a sizeable impact. Following [13, 14], we write

$$\left[\frac{\Phi_{\rho\gamma}}{\Phi_{K^*\gamma}}\right]\left|\frac{\mathcal{P}_{tc}^{\rho\gamma}}{\mathcal{P}_{tc}^{K^*\gamma}}\right|^2=\left[\frac{M_B^2-M_\rho^2}{M_B^2-M_{K^*}^2}\right]^3\zeta^2, \quad (19)$$

where $\zeta=F_\rho/F_{K^*}$ is the $SU(3)$-breaking ratio of the $B^{\pm}\to\rho^{\pm}\gamma$ and $B^{\pm}\to K^{*\pm}\gamma$ form factors; a light-cone sum-rule analysis gives $\zeta^{-1}=1.31\pm0.13$ [23], to be compared with the result $\zeta^{-1}=1.1\pm0.1$ of a preliminary lattice analysis [24]. Consequently, (18) and (19) allow us to convert the measured $B^{\pm}\to K^{*\pm}\gamma$ branching ratio (11) into a *lower* SM bound for ${\rm BR}(B^{\pm}\to\rho^{\pm}\gamma)$ with the help of (14). If we use the SM range $\gamma=(65\pm7)^\circ$ [6] and $\zeta^{-1}=1.31\pm0.13$ [23], we obtain

$${\rm BR}(B^{\pm}\to\rho^{\pm}\gamma)_{\rm min}=\left(1.02\,^{+0.27}_{-0.23}\right)\times10^{-6}. \quad (20)$$

A similar kind of reasoning holds also for the U-spin pairs $B^{\pm}\to K^{\pm}K,\pi^{\pm}K$ and $B^{\pm}\to K^{\pm}K^*,\pi^{\pm}K^*$. In the former case, the factorizable $SU(3)$-breaking effects are governed by (4). In the latter case, following [8], the form factors $f_+^{K,\pi}$ enter. However, because of $f_+^P=f_0^P$,

we arrive again at (4). Using then the experimental results $\mathrm{BR}(B^\pm \to \pi^\pm K) = (24.1 \pm 1.3) \times 10^{-6}$ and $\mathrm{BR}(B^\pm \to \pi^\pm K^*) = (9.76\,^{+1.16}_{-1.22}) \times 10^{-6}$ [11], we obtain

$$\mathrm{BR}(B^\pm \to K^\pm K)_{\rm min} = \Xi^K_\pi \times \left(1.69\,^{+0.21}_{-0.24}\right) \times 10^{-6} \quad (21)$$
$$\mathrm{BR}(B^\pm \to K^\pm K^*)_{\rm min} = \Xi^K_\pi \times \left(0.68\,^{+0.11}_{-0.13}\right) \times 10^{-6}. \quad (22)$$

In the case of $B^\pm \to K^\pm K$, the lower SM bound is very close to the experimental upper bound of 2.4×10^{-6} [9], whereas the upper limit of 5.3×10^{-6} for $B^\pm \to K^\pm K^*$ still leaves a lot of space. Obviously, we may also consider the $B^\pm \to K^{*\pm} K, \rho^\pm K$ system. Using the $B \to V$ form factors obtained in [23] to deal with the factorizable $SU(3)$-breaking effects, we arrive at

$$\frac{\mathrm{BR}(B^\pm \to K^{*\pm} K)}{\mathrm{BR}(B^\pm \to \rho^\pm K)} \geq \Xi^{K^*}_\rho \times 0.084 \times \left[1 - 2\epsilon \cos^2\gamma\right] \sin^2\gamma, \quad (23)$$

with

$$\Xi^{K^*}_\rho = \left[\frac{A_0^{K^*}}{0.470} \frac{0.372}{A_0^\rho}\right]^2. \quad (24)$$

Although the individual form factors are very different, (24) yields essentially the same correction as (4). Since only the upper bound of $\mathrm{BR}(B^\pm \to \rho^\pm K) < 48 \times 10^{-6}$ is available, we may not yet apply (23).

Let us now turn to $\bar{B}^0_d \to \rho^0 \gamma$, which receives contributions from exchange and penguin annihilation topologies that are not present in $\bar{B}^0_d \to \bar{K}^{*0}\gamma$; in the case of $B^\pm \to \rho^\pm \gamma$ and $B^\pm \to K^{*\pm}\gamma$, which are related by the U-spin symmetry, there is a one-to-one correspondence of topologies [20]. Making the plausible assumption that the topologies involving the spectator quarks play a minor rôle, and taking the factor of $c_{\rho^0} = 1/\sqrt{2}$ in (9) into account, the counterpart of (20) is given by

$$\mathrm{BR}(B_d \to \rho^0 \gamma)_{\rm min} = \left(0.51\,^{+0.13}_{-0.11}\right) \times 10^{-6}. \quad (25)$$

If we compare the *lower* SM bounds in (20) and (25) with the current experimental *upper* bounds in (5) and (6), respectively, we observe that they are remarkably close to one another. Consequently, we expect that the $\bar{B} \to \rho\gamma$ modes should soon be discovered at the B factories. The next important step would then be the measurement of their CP-violating observables.

The authors of [13, 14] followed a different avenue to confront the experimental bounds in (5) and (6) with the SM, converting them into upper bounds for the side $R_t \propto |V_{td}/V_{cb}|$ of the unitarity triangle [19]. To this end, they use also (19), and calculate the CP-conserving (complex) parameter δa entering $\rho_{\rho\gamma} e^{i\theta_{\rho\gamma}} = R_b\,[1 + \delta a]$ with the help of QCD factorization. The corresponding result, which favours a small impact of δa, takes leading and next-to-leading order QCD corrections into account and holds to leading order in the heavy-quark limit [14]. However, in view of the remarks about possible long-distance effects made above and the B-factory data for the $B \to \pi\pi$ system, which indicate large corrections to the QCD factorization picture for non-leptonic B decays into two light pseudoscalar mesons [7, 25], it is not obvious to us that the impact of δa is actually small. The advantage of our bound following from (18) is that it is – by construction – *not* affected by $\rho_{\rho\gamma} e^{i\theta_{\rho\gamma}}$ at all.

Interestingly, the lower bounds for the CP-averaged $B^\pm \to K^{(*)\pm} K^{(*)}$ and $B \to \rho\gamma$ branching ratios discussed above are actually realizations of a general bound that can be derived in the SM for $b \to d$ penguin processes. Let us consider such a decay, $\bar{B} \to \bar{f}_d$. In analogy to (1) and (9), we may then write

$$A(\bar{B} \to \bar{f}_d) = A_d^{(0)} \left[1 - \rho_d e^{i\theta_d} e^{-i\gamma}\right], \quad (26)$$

so that the CP-averaged amplitude square takes the form

$$\langle |A(B \to f_d)|^2 \rangle = |A_d^{(0)}|^2 \left[1 - 2\rho_d \cos\theta_d \cos\gamma + \rho_d^2\right]. \quad (27)$$

In general, ρ_d and θ_d depend on the point in phase space considered. This has the implication that the expression

$$\mathrm{BR}(B \to f_d) = \tau_B \left[\sum_{\rm Pol} \int d\,\mathrm{PS}\, \langle |A(B \to f_d)|^2 \rangle\right] \quad (28)$$

for the CP-averaged branching ratio, where the sum runs over possible polarization configurations of f_d, does *not* factorize into $|A_d^{(0)}|^2$ and $[1 - 2\rho_d \cos\theta_d \cos\gamma + \rho_d^2]$ as in the case of the two-body decays considered above. However, if we keep ρ_d and θ_d as free, "unknown" parameters at any given point in phase space, we obtain

$$\langle |A(B \to f_d)|^2 \rangle \geq |A_d^{(0)}|^2 \sin^2\gamma, \quad (29)$$

which implies

$$\mathrm{BR}(B \to f_d) \geq \tau_B \left[\sum_{\rm Pol} \int d\,\mathrm{PS}\, |A_d^{(0)}|^2\right] \sin^2\gamma. \quad (30)$$

We consider now a $b \to s$ penguin process $\bar{B} \to \bar{f}_s$, which is the counterpart of $\bar{B} \to \bar{f}_d$ in that the corresponding CP-conserving strong amplitudes can be related to one another through the $SU(3)$ flavour symmetry. In analogy to (13), we then have

$$A(\bar{B} \to \bar{f}_s) = -\frac{A_s^{(0)}}{\sqrt{\epsilon}} \left[1 + \epsilon \rho_s e^{i\theta_s} e^{-i\gamma}\right]. \quad (31)$$

If we neglect the term proportional to ϵ in the square bracket, we arrive at

$$\frac{\mathrm{BR}(B \to f_d)}{\mathrm{BR}(B \to f_s)} \geq \epsilon \left[\frac{\sum_{\rm Pol} \int d\,\mathrm{PS}\, |A_d^{(0)}|^2}{\sum_{\rm Pol} \int d\,\mathrm{PS}\, |A_s^{(0)}|^2}\right] \sin^2\gamma. \quad (32)$$

Apart from the tiny ϵ correction, which gave a shift of about 1.9% in (18), (32) is valid exactly in the SM. If we now apply the $SU(3)$ flavour symmetry, we obtain

$$\frac{\sum_{\rm Pol} \int d\,\mathrm{PS}\, |A_d^{(0)}|^2}{\sum_{\rm Pol} \int d\,\mathrm{PS}\, |A_s^{(0)}|^2} \stackrel{SU(3)_{\rm F}}{\longrightarrow} 1. \quad (33)$$

Since, in the SM, $\sin^2\gamma \sim 0.8$ is favourably large and the decay $\bar{B} \to \bar{f}_s$ will be measured before its $b \to d$ counterpart – simply because of the CKM enhancement – (32) provides strong lower bounds for $\mathrm{BR}(B \to f_d)$.

It is instructive to return briefly to $B \to \rho\gamma$. Looking at (32), we observe immediately that the assumption that these modes are governed by a single photon helicity is no longer required. Consequently, (20) and (25) are actually very robust with respect to this issue, which may only affect the $SU(3)$-breaking corrections to a small extend.

We may now also complement the bounds in (21)–(23) through the $B^\pm \to K^{*\pm}K^*, \rho^\pm K^*$ system, where we have to sum in (32) over three polarization configurations of the vector mesons. The analysis of the $SU(3)$-breaking corrections is now more involved. However, if we expand in M_ρ/M_B and M_{K^*}/M_B, and use the form-factor relation $A_3^V = A_0^V$ [23], we find that the factorizable corrections are described to a good approximation by (24). Using then the very recent result of $\mathrm{BR}(B^\pm \to \rho^\pm K^*) = (9.2 \pm 2.0) \times 10^{-6}$ [11], we obtain

$$\mathrm{BR}(B^\pm \to K^{*\pm}K^*)_{\min} \approx \Xi_\rho^{K^*} \times \left(0.64^{+0.15}_{-0.16}\right) \times 10^{-6}; \quad (34)$$

the current experimental upper bound reads 71×10^{-6}. Interestingly, (34) would be reduced by ~ 0.6 in the strict $SU(3)$ limit, i.e. would be more conservative. A similar comment applies to (2), (3) and (21)–(23). On the other hand, the $B \to \rho\gamma$ bounds in (20) and (25) would be enhanced by ~ 1.7 in this case. However, here the theoretical situation is more favourable since we have not to rely on the factorization hypothesis to deal with the $SU(3)$-breaking effects as in the non-leptonic decays [5].

Another interesting application of (32) is offered by decays of the kind $\bar{B} \to \pi\ell^+\ell^-$ and $\bar{B} \to \rho\ell^+\ell^-$. It is well known that the ρ_d terms complicate the interpretation of the corresponding data considerably [21]; our bound offers SM tests that are not affected by these contributions. The structure of the $b \to d\ell^+\ell^-$ Hamiltonian is similar to (7), but involves the additional operators

$$Q_{9,10} = \frac{\alpha}{2\pi}(\bar{\ell}\ell)_{\mathrm{V,A}}(\bar{d}_i b_i)_{\mathrm{V-A}}. \quad (35)$$

The $b \to s$ counterparts of these transitions, $\bar{B} \to K\ell^+\ell^-$ and $\bar{B} \to K^*\ell^+\ell^-$, were already observed at the B factories, with branching ratios at the 0.6×10^{-6} and 1.4×10^{-6} levels [11], respectively, and received a lot of theoretical attention (see, for instance, [26]). For the application of (32), the charged decay combinations $B^\pm \to \pi^\pm\ell^+\ell^-, K^\pm\ell^+\ell^-$ and $B^\pm \to \rho^\pm\ell^+\ell^-, K^{*\pm}\ell^+\ell^-$ are suited best since the corresponding decay pairs are related to each other through the U-spin symmetry [22]. We strongly encourage detailed studies of the associated $SU(3)$-breaking corrections to (33) and are confident that we will have a good picture of these effects by the time the $B^\pm \to \pi^\pm\ell^+\ell^-$, $\rho^\pm\ell^+\ell^-$ modes will come within experimental reach.

Should the $b \to d$ penguin decays actually be found in accordance with the bounds derived above, as in the case of $\bar{B}_d^0 \to K^0\bar{K}^0$, we would have a first confirmation of the SM description of the corresponding FCNC processes. On the other hand, it would be much more exciting if some bounds should be significantly violated through the destructive interference between NP and SM contributions. As the various decay classes are governed by different operators, we may well encounter surprises.

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[1] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
[2] R. Fleischer, *Phys. Lett.* **B 341**, 205 (1994).
[3] A. J. Buras, R. Fleischer and T. Mannel, *Nucl. Phys.* **B 533**, 3 (1998).
[4] M. Beneke, M. Neubert, *Nucl. Phys.* **B 675**, 333 (2003).
[5] R. Fleischer and S. Recksiegel, CERN-PH-TH/2004-146 [hep-ph/0408016], to appear in *Eur. Phys. J.* **C**.
[6] M. Battaglia *et al.*, hep-ph/0304132.
[7] A. J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Phys. Rev. Lett.* **92**, 101804 (2004); *Nucl. Phys.* **B 697**, 133 (2004).
[8] P. Ball and R. Zwicky, hep-ph/0406232.
[9] B. Aubert *et al.* [BaBar Collaboration], hep-ex/0408080.
[10] 32nd International Conference on High-Energy Physics, Beijing, China, August 2004, http://ichep04.ihep.ac.cn/.
[11] http://www.slac.stanford.edu/xorg/hfag/.
[12] A. K. Giri and R. Mohanta, hep-ph/0408337.
[13] A. Ali, E. Lunghi and A. Y. Parkhomenko, *Phys. Lett.* **B 595**, 323 (2004).
[14] S. W. Bosch and G. Buchalla, hep-ph/0408231.
[15] B. Aubert *et al.* [BaBar Collaboration], hep-ex/0408034.
[16] K. Abe *et al.* [Belle Collaboration], hep-ex/0408137.
[17] G. Buchalla, A. J. Buras and M. E. Lautenbacher, *Rev. Mod. Phys.* **68**, 1125 (1996).
[18] L. Wolfenstein, *Phys. Rev. Lett.* **51**, 1945 (1983).
[19] A. J. Buras, M. E. Lautenbacher and G. Ostermaier, *Phys. Rev.* **D 50**, 3433 (1994).
[20] B. Grinstein, D. Pirjol, *Phys. Rev.* **D 62**, 093002 (2000).
[21] P. Ball *et al.*, hep-ph/0003238.
[22] T. Hurth and T. Mannel, *Phys. Lett.* **B 511**, 196 (2001).
[23] P. Ball, V. M. Braun, *Phys. Rev.* **D 58**, 094016 (1998).
[24] D. Bećirević, talk at the Ringberg Workshop on Heavy Flavours, Ringberg Castle, Tegernsee, May 2003.
[25] A. Ali, E. Lunghi and A. Y. Parkhomenko, *Eur. Phys. J.* **C 36**, 183 (2004); C. W. Chiang, M. Gronau, J. L. Rosner and D. A. Suprun, *Phys. Rev.* **D 70**, 034020 (2004); C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, hep-ph/0401188.
[26] A. Ali, P. Ball, L. T. Handoko and G. Hiller, *Phys. Rev.* **D 61**, 074024 (2000); M. Beneke, T. Feldmann and D. Seidel, *Nucl. Phys.* **B 612**, 25 (2001); A. Ali and A. S. Safir, *Eur. Phys. J.* **C 25**, 583 (2002).